XII HSC - BOARD - MARCH - 2017

Date: 06.03.2017

MATHEMATICS (40) - SOLUTIONS

The following scheme of marking is only for guidelines to help to evaluate answer papers. Any alternative, but logically correct approach, should be acceptable and must be given full credit. Part marking should be made strictly according to the number of correct steps.

SECTION - I

Q. 1 (A)

(i) (c) 4 **[2 M]**

solution:

Direction Ratio of $\overline{AB} = (-2,-2,3)$

Direction Ratio of $\overline{BC} = (k,4,-6)$

If point A,B and C are collinear then $\dfrac{DR \text{ of } \overline{AB}}{DR \text{ of } \overline{BC}} = \text{constant}$

$$\frac{-2}{k} = \frac{-2}{4}$$

$$\boxed{k=4}$$

Topic:3-D Geometry_; Sub-topic:Direction ratios and direction cosins_ L- 2_Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) (a) $\dfrac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ **[2 M]**

Solution:

$|A| = -2+15=13$

$$A^{-1} = \frac{adj\ A}{|A|} = \frac{\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}}{13}$$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Topic:Matrices; Sub-topic:Inverse of Matrix _ L-1 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) (b) $\frac{1}{\sqrt{5}}$ **[2 M]**

Solution:

$a = 13, b=14, c=15$

$$s = \frac{13+14+15}{2} = 21$$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{b\times c}}$$

$$\sin\frac{A}{2} = \sqrt{\frac{(21-14)(21-15)}{14\times 15}}$$

$$\sin\frac{A}{2} = \sqrt{\frac{7\times 6}{14\times 15}}$$

$$\sin\frac{A}{2} = \frac{1}{\sqrt{5}}$$

Topic:Trigonometric Function; Sub-topic:Solution of Triangle _ L-1_Target-2017_XII-HSC Board (40) Test_Mathematics

(B)

(i) If $\vec{a}\ \vec{b}\ \&\ \vec{c}$ are conterminus edges of parallelopiped then the volume of the parallelopiped

$= \left[\vec{a}\ \vec{b}\ \vec{c}\right]$

where $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{b} = 5\hat{i} + 7\hat{j} + 5\hat{k}$

$\vec{c} = 4\hat{i} + 5\hat{j} - 2\hat{k}$

$$\therefore \quad V = \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 2 & 3 & -4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{vmatrix}$$ **[1 M]**

$= 2(-14-25) - 3(-10-20) - 4(25-28)$

$= 2(-39) - 3(-30) - 4(-3)$

$= -78 + 90 + 12$

$= 24$ cube unit **[1 M]**

Topic: Vectors_Subtopic_Scalar triple product_L-1__Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) Taking LHS

$ab\cos C - ac\cos B$

$$= ab\left(\frac{a^2+b^2-c^2}{2ab}\right) - ac\left(\frac{a^2+c^2-b^2}{2ac}\right)$$ **[1 M]**

$$= \frac{a^2+b^2-c^2-a^2-c^2+b^2}{2} = \frac{2b^2-2c^2}{2} = b^2-c^2 = RHS$$ **[1 M]**

Topic: Trigonometric function__Subtopic_SOT_L-1_Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) QA and QB are the perpendiculars drawn from the point $Q(a,b,c)$ to YZ and ZX planes.

$\therefore\ A = (0, b, c)$ *and* $B = (a, 0, c)$

The required plane is pasing through O(0, 0, 0), A(0, b, c) and B(a, 0, c)

The vector equation of the plane passing thorugh the O, A, B is

$$\bar{r}\cdot\left(\overline{OA}\times\overline{OB}\right) = \bar{0}\cdot\left(\overline{OA}\times\overline{OB}\right)$$

i.e., $\bar{r}\cdot\left(\vec{a}\times\vec{b}\right) = 0$ **[1 M]**

Now, $\overline{OA} = \bar{a} = 0.\hat{i}+b\hat{j}+c\hat{k}$

and $\overline{OB} = \bar{b} = a\hat{i} + 0.\hat{j} + c\hat{k}$

$$\therefore\ \overline{OA}\times\overline{OB} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & b & c\\ a & 0 & c\end{vmatrix}$$

$$= (bc-0)\hat{i} - (0-ac)\hat{j} + (0-ab)\hat{k}$$

$$= bc\hat{i} + ac\hat{j} - ab\hat{k}$$

$\therefore$ from (1), the vector equation of the required plane is

$$\bar{r}\cdot\left(bc\hat{i}+ac\hat{j}-ab\hat{k}\right) = 0$$ **[1 M]**

Topic: Plane_Subtopic_Equation of Plane_L-1_Target-2017_XII-HSC Board (40) Test_Mathematics

(iv) Equation of line passing through the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$ **[1 M]**

Equation of line passing through the point $A(3,4,-7)$ and $B(6,-1,1)$ is

$$\frac{x-3}{6-3} = \frac{y-4}{-1-4} = \frac{z-(-7)}{1-(-7)}$$

$$\frac{x-3}{3} = \frac{y-4}{-5} = \frac{z+7}{8}$$ **[1 M]**

Topic_Line_subtopic_equation of line _L-1_Target-2017_XII-HSC Board (40) Test_Mathematics

(v) Let $P \equiv \forall\, n \in N, n^2 + n$ is an even number

$q \equiv \forall\, n \in N, n^2 - n$ is an odd number

The symbolic form of given statement is

$(p \wedge q)$ **[1 M]**

Truth value of given statement is

$P \equiv \forall\, n \in N, n^2 + n$ is an even number (T)

$q \equiv \forall\, n \in N, n^2 - n$ is an odd number (F)

($\because$ from n = 1 , $n^2 - n = 0,$ which is not an odd number)

$\therefore (p \wedge q) \equiv T \wedge F \equiv F$

$\therefore$ given statement is false **[1 M]**

Topic:Logic_; Sub-topic:Truth values_ L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

Q. 2 (A)

(i) No of rows = $2^n = 2^3 = 8$

No. of columns = $m + n = 3 + 3 = 6$ **[1 M]**

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

[1 M]

In the last column, the truth values of the statement is neither all T nor all F.
Hence, it is neither a tautology nor a contradiction i.e. it is a contingency. **[1 M]**

Topic:Logic; Sub-topic:_Statement pattern _ L-2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) The lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \qquad ....(i)$$

and $$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \qquad ....(ii)$$

Here $x_1 = 1,\ y_1 = 2,\ z_1 = 3,$ $\qquad x_2 = 2,\ y_2 = 4,\ z_2 = 5$

$a_1 = 2,\ b_1 = 3,\ c_1 = 4,$ $\qquad a_2 = 3,\ b_2 = 4,\ c_2 = 5$

Shortest distance between the lines is

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$ **[1 M]**

Now $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

$= 1(15-16) - 2(10-12) + 2(8-9)$

$= -1 + 4 - 2$

$= 1$ **[1 M]**

and $(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2 = (15-16)^2 + (12-10)^2 + (8-9)^2$

$= 1 + 4 + 1$

$= 6$

Hence, the shortest distance between the lines (i) and (ii) is

$= \left| \frac{1}{\sqrt{6}} \right|$

$= \frac{1}{\sqrt{6}}$ units **[1 M]**

Topic:Line; Sub-topic:Distance between line_L-2__Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) $(\sin 2x + \sin 6x) + \sin 4x = 0$

$2\sin 4x \cdot \cos 2x + \sin 4x = 0$

$\sin 4x[2\cos 2x + 1] = 0$

$\sin 4x = 0$ or $2\cos 2x + 1 = 0$ **[1 M]**

$\sin 4x = 0$, or $\cos 2x = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$

using $\sin x = 0 \Rightarrow x = n\pi$

$\therefore \sin 4x = 0$

$\therefore 4x = n\pi$,

using $\cos x = \cos\alpha \Rightarrow x = 2mx \pm \alpha$

$\cos 2x = \cos\frac{2\pi}{3}$

$2x = 2m\pi \pm \frac{2\pi}{3}$

$\therefore$ The general solution is x

$x = \frac{n\pi}{4}$ **[1 M]**

$x = m\pi \pm \frac{\pi}{3}$ where $m, n \in z$ **[1 M]**

Topic:_Trigonometric function; Sub-topic:_Solutions of equation _ L-3 _Target-2017_XII-HSC Board (40) Test_Mathematics

(B)

(i) x – y + z = 4
2x + y – 3z = 0
x + y + z = 2

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$ **[1 M]**

$R_2 - 2R_1$ & $R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}$$ **[1 M]**

x – y + z = 4 .... (1)
3y – 5z = – 8 .... (2)
2y = – 2 .... (3) **[1 M]**
∴ y = –1
By equation (2)
–3 – 5z = – 8
–5z = –5
z = 1
∴ By equation (1)
x + 1 + 1 = 4
x = 2
Ans : x = 2, y = –1, z = 1 **[1 M]**

Topic:Matrix; Sub-topic:_Application of matrix _ L- 2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) Let m_1 and m_2 be the slopes of the lines represented by the equation

$ax^2 + 2hxy + by^2 = 0$ ......(1)

Then their separate equations are

$y = m_1x$ and $y = m_2x$

∴ Then their combined equation is

$(m_1x - y)(m_2x - y) = 0$

i.e, $m_1m_2x^2 - (m_1 + m_2)\, xy + y^2 = 0$ .....(2)

Since (1) and (2) represent the same two lines, comparing the coefficients, we get,

$$\frac{m_1m_2}{a} = \frac{-(m_1+m_2)}{2h} = \frac{1}{b}$$

$\therefore m_1 + m_2 = -\frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$ **[1 M]**

$\therefore (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$$=\frac{4h^2}{b^2}-\frac{4a}{b}=\frac{4\left(h^2-ab\right)}{b^2}$$

$$\therefore \left|m_1-m_2\right|=\left|\frac{2\sqrt{h^2-ab}}{b}\right|$$ **[1 M]**

If θ is the acute angle between the lines, then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|, \text{if } m_1m_2\neq -1$$

$$=\left|\frac{\left(2\sqrt{h^2-ab}\right)/b}{1+\left(a/b\right)}\right|, \text{ if } \frac{a}{b}\neq -1$$

$$\therefore \tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|, \text{ if } a+b\neq 0$$ **[1 M]**

For coincident lines, $\theta=0 \therefore \tan\theta=0 \therefore h^2=ab$ **[1 M]**

Topic:Pair of straight line ; Sub-Topic:Combined equation of two lines_L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) Let $\overline{p},\overline{q},\overline{r}$ be the position vectors of vertices P, Q, R of ΔPQR respectively

$$\overline{p}=4\hat{j},\overline{q}=3\hat{k},\overline{r}=4\hat{j}+3\hat{k}$$

$$\overline{PQ}=\overline{q}-\overline{p}=3\hat{k}-4\hat{j}=-4\hat{j}+3\hat{k}$$

$$\overline{QR}=\overline{r}-\overline{q}=4\hat{j}+3\hat{k}-3\hat{k}=4\hat{j}$$

$$\overline{RP}=\overline{p}-\overline{r}=4\hat{j}-4\hat{j}-3\hat{k}=-3\hat{k}$$ **[1 M]**

Let x,y,z be the lengths of opposites of vertices P,Q,R respectively.

$$x=\left|\overline{QR}\right|=4 \qquad y=\left|\overline{RP}\right|=3$$

$$z=\left|\overline{PQ}\right|=\sqrt{16+9}=\sqrt{25}=5$$ **[1 M]**

If $H\left(\overline{h}\right)$ is the incentre of ΔPQR then

$$\overline{h}=\frac{x\overline{p}+y\overline{q}+z\overline{r}}{x+y+z}$$

$$=\frac{4\left(4\hat{j}\right)+3\left(3\hat{k}\right)+5\left(4\hat{j}+3\hat{k}\right)}{4+3+5}$$ **[1 M]**

$$=\frac{16\hat{j}+9\hat{k}+20\hat{j}+15\hat{k}}{12}$$

$$=\frac{36\hat{j}+24\hat{k}}{12}=3\hat{j}+2\hat{k}$$ **[1 M]**

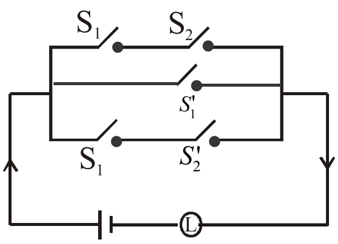
Topic:Vector; Sub-topic:Geometrical application_L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

Q. 3 (A)

(i) Let $p \equiv$ Switch S_1 is closed

$q =$ Switch S_2 is closed **[1 M]**

$\therefore \sim p =$ Switch S_1' & $\sim q \equiv S_2'$ **[1 M]**



[1 M]

Topic:Logic; Sub-topic:Application of logic _ L-2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) Comparing the equation $5x^2 + 2xy - 3y^2 = 0$, we get,

$a = 5$, $2h = +2$, $b = -3$

Let m_1 and m_2 be the slopes of the lines represented by $5x^2 + 2xy - 3y^2 = 0$

$$\therefore m_1 + m_2 = \frac{-2h}{b} = \frac{-2}{3} \qquad ....(1)$$

and $m_1 m_2 = \dfrac{a}{b} = \dfrac{+5}{-3}$

Now required lines are perpendicular to these lines

$\therefore$ their slopes are $-1/m_1$ and $-1/m_2$ **[1 M]**

Since these lines are passing through the origin, their separate equations are

$$y = \frac{-1}{m_1}x \text{ and } y = \frac{-1}{m_2}x$$

i.e., $m_1 y = -x$ and $m_2 y = -x$

i.e., $x + m_1 y = 0$ and $x + m_2 y = 0$

$\therefore$ their combined equation is

$(x + m_1 y)(x + m_2 y) = 0$

$\therefore x^2 + (m_1 + m_2)xy + m_1 m_2 y_2 = 0$ **[1 M]**

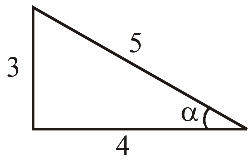
$$\therefore x^2 + \frac{-2}{3}xy + \frac{-5}{3}y^2 = 0$$

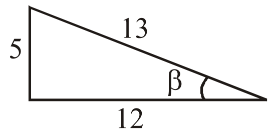
$3x^2 - 2xy - 5y^2 = 0$ **[1 M]**

Topic:Pair of St.Lines; Sub-Topic:Combined homogeneous equations_L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) Let $\cos^{-1}\left(\frac{4}{5}\right)=\alpha$ $\qquad$ $\cos^{-1}\left(\frac{12}{13}\right)=\beta$

$\therefore \cos\alpha = \frac{4}{5}$ $\qquad$ $\cos\beta = \frac{12}{13}$





$\sin\alpha=\frac{3}{5}$ $\qquad$ $\sin\beta=\frac{5}{13}$ **[1 M]**

Using,

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta$$

$$=\frac{4}{5}\cdot\frac{12}{13}-\frac{3}{5}\cdot\frac{5}{13}$$

$$=\frac{48-15}{65}=\frac{33}{65}$$ **[1 M]**

$$\therefore \alpha+\beta=\cos^{-1}\left(\frac{33}{65}\right)$$

$$\therefore \cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$$

Hence proved **[1 M]**

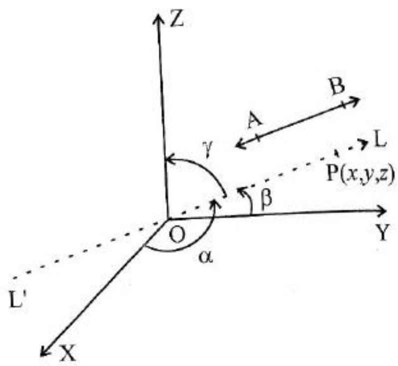
Topic: Trigonometric functions ; Sub-Topic:Inverse Trigonometric functions_L-2__Target-2017_XII-HSC Board (40) Test_Mathematics

(B)

(i) Let α, β, γ be the angles made by the line with X-, Y-, Z- axes respectively.

$\therefore l=\cos\alpha,\ m=\cos\beta$ and $n=\cos\gamma$

Let $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ be any non-zero vector along the line.



Since $\hat{i}$ is the unit vector along X-axis,

$$\bar{a}\cdot\hat{i}=|\bar{a}|\cdot|\hat{i}|\cos\alpha=a\cos\alpha$$

Also, $\bar{a}\cdot\hat{i}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})\cdot\hat{i}$

$$=a_1\times1+a_2\times0+a_3\times0=a_1$$

$\therefore a\cos\alpha=a_1$ $\qquad$ ...(1) **[1 M]**

Since $\hat{j}$ is the unit vector along Y-axis,

$$\bar{a}\cdot\hat{j}=|\bar{a}|\cdot|\hat{j}|\cos\beta=a\cos\beta$$

Also, $\bar{a}\cdot\hat{j}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})\cdot\hat{j}$

$$=a_1\times0+a_2\times1+a_3\times0=a_2$$

$\therefore a\cos\beta = a_2$ ...(2)

Similarly, $a\cos\gamma = a_3$ ...(3) **[1 M]**

$\therefore$ from equations (1), (2) and (3),

$a^2\cos^2\alpha + a^2\cos^2\beta + a^2\cos^2\gamma = a_1{}^2 + a_2{}^2 + a_3{}^2$

$\therefore a^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = a^2$ ...$[\because a = |\bar{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}]$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(I) $[\because a \neq 0]$

i.e., $l^2 + m^2 + n^2 = 1$. **[1 M]**

Also

$\alpha = ?,\ \beta = 135°,\ \gamma = 45°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 45°$

$\cos^2\alpha + \cos^2 135° + \cos^2 45° = 1$

$\cos^2\alpha + \frac{1}{2} + \frac{1}{2} = 1$

$\cos^2\alpha = 0$

$\therefore \alpha = \frac{\pi}{2}$ OR $\frac{3\pi}{2}$ **[1 M]**

Topic:3D; Sub-topic:_direction cosines _ L-1 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) The vector equation of the plane passing through the points $A(\bar{a}), B(\bar{b})$ *and* $C(\bar{c})$

$\bar{r}\cdot(\overline{AB}\times\overline{AC}) = \bar{a}\cdot(\overline{AB}\times\overline{AC})$.......(1)

Let $\bar{a} = \hat{i} + \hat{j} - 2\hat{k}, \bar{b} = \hat{i} + 2\hat{j} + \hat{k}, \bar{c} = 2\hat{i} - \hat{j} + \hat{k}$

$\therefore \overline{AB} = \bar{b} - \bar{a} = (\hat{i} + 2\hat{j} + \hat{k}) - (\hat{i} + \hat{j} - 2\hat{k}) = \hat{j} + 3\hat{k}$

and $\overline{AC} = \bar{c} - \bar{a} = (2\hat{i} - \hat{j} + \hat{k}) - (\hat{i} - \hat{j} - 2\hat{k}) = \hat{i} - 2\hat{j} + 3\hat{k}$ **[1 M]**

$$\therefore \overline{AB}\times\overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 3 \\ 1 & -2 & 3 \end{vmatrix}$$

$= (3+6)\hat{i} - (0-3)\hat{j} + (0-1)\hat{k}$

$= 9\hat{i} + 3\hat{j} - \hat{k}$ **[1 M]**

and $\bar{a}\cdot(\overline{AB}\times\overline{AC}) = (\hat{i} + \hat{j} - 2\hat{k})\cdot(9\hat{i} + 3\hat{j} - \hat{k})$

$=1(9)+1(3)+(-2)(-1)$

$=9+3+2=14$ **[1 M]**

$\therefore$ from (1), the vector equation of the required plane is

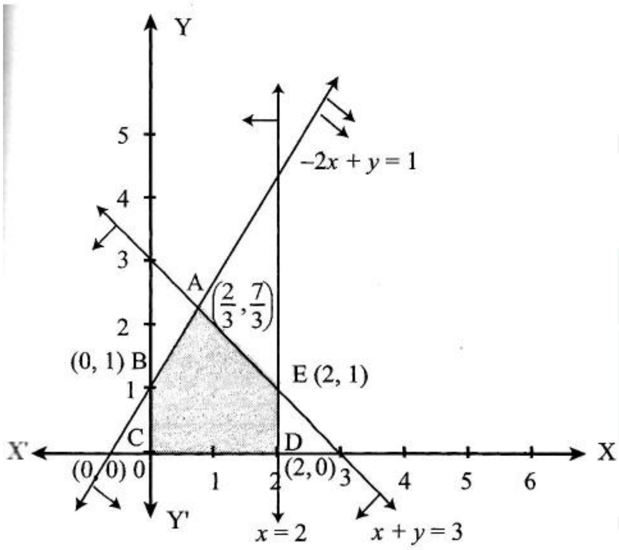
$\bar{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$ **[1 M]**

Topic:Plane; Sub-topic:Equation of plane _ L- 2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) Let $x=2$, $x+y=3$, $-2x+y=1$

$x=0, y=3;(0,3)$ $\quad$ $x=0, y=1;(0,1)$

$y=0, x=3;(3,0)$ $\quad$ $x=1, y=3;(1,3)$ **[1 M]**



Scale 1 unit = 1 cm on both axis **[1 M]**

$\therefore$ ABCDEA is the feasible region

From the above figure by solving the points are A,B,C,D,E where

$A\left(\frac{2}{3},\frac{7}{3}\right)$; B(0,1); C(0,0); D(2,0) $\quad ,E(2,1)$ **[1 M]**

End points	Value of $z=6x+4y$
$A\left(\frac{2}{3},\frac{7}{3}\right)$	$6\left(\frac{2}{3}\right)+4\left(\frac{7}{3}\right)=\frac{12+28}{3}=\frac{40}{3}=13.33$
$B(0,1)$	$0+4=4$
$C(0,0)$	$0+0=0$
$D(2,0)$	$12+0=12$
$E(2,1)$	$12+4=16$

$\therefore z$ is maximum 16 at the point (2,1) **[1 M]**

Topic:LPP; Sub-topic:Graphical solution_L-2_XII-HSC Board (40) Test_Mathematics

SECTION - II

Q. 4 (A)

(i) (b) $\frac{\sqrt{3}}{2}$ **[2 M]**

Solution Let $y=\tan^3\theta$, and $x=\sec^3\theta$

$$\frac{dy}{d\theta}=3\tan^2\theta\cdot\sec^2\theta,\frac{dx}{d\theta}=3\sec^2\theta\cdot\sec\theta\tan\theta$$

$$\frac{dy}{dx}=\sin\theta$$

$$=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

Topic:Differentation; Sub-topic:Parametric form _ L-1 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) (a) $5x-y=2$ **[2 M]**

Solution $y=3x^2-x+1$

$$\frac{dy}{dx}=6x-1$$

$$\text{slope of tangent}=\left[\frac{dy}{dx}\right]_{at(1,3)}=6\times1-1=5$$

$$(x_1,y_1)=(1,3)$$

$$\text{equation of tangent}\Rightarrow y-y_1=m(x-x_1)$$

$$\Rightarrow y-3=5(x-1)$$

$$\Rightarrow y-3=5x-5$$

$$\Rightarrow 5x-y=2$$

Topic:Application of Derivative; Sub-topic:Tangent and Normal _ L-2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) (b) 1.5 **[2 M]**

Solution $p=\frac{1}{2}$

$q=\frac{1}{2}$ $\quad\because[q=1-(p)]$

$n=3$

$$\text{Expected value E(X)}=np=3\times\frac{1}{2}=1.5$$

Topic: Probability distribution_Sub Topic:Expected Mean_Level: 1_Target-2017_XII-HSC Board (40) Test_Mathematics

(B)

(i) $x\sin y + y\sin x = 0$

Differentiate w.r.t. x both side

$$\left[x\cos y\frac{dy}{dx}+\sin y\right]+\left[y\cos x+\sin x\frac{dy}{dx}\right]=0$$ **[1 M]**

$$\therefore \sin y + y\cos x = \frac{dy}{dx}(-\sin x - x\cos y)$$

$$\therefore \frac{dy}{dx} = -\left(\frac{\sin y + y\cos x}{\sin x + x\cos y}\right)$$ **[1 M]**

Topic: Differentiation_Sub Topic: Implicit Function_Level: 1_Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) $f(x) = x - \dfrac{1}{x}, x \in R$

$$\therefore f'(x) = 1 - \left(\frac{-1}{x^2}\right) = 1 + \frac{1}{x^2}$$ **[1 M]**

$\because x \neq 0$, for all values of x, $x^2 > 0$

$\therefore \dfrac{1}{x^2} > 0, \quad \therefore 1+\dfrac{1}{x^2}$ is always positive

Thus, $f'(x) > 0$, for all $x \in R$

Hence $f(x)$ is increasing function. **[1 M]**

Topic:Application of Derivative; Sub-topic:Increasing and Decreasing function _ L-1 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) Let $I = \displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

Let $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\frac{1}{\sqrt{x}}dx = 2\,dt$$ **[1 M]**

$$\therefore I = 2\int \sin t \cdot dt$$

$$= -2\cos t + C$$

$$= -2\cos\left(\sqrt{x}\right) + C$$ **[1 M]**

Topic:Integration; Sub-topic:Method of Substitution_ L-1 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iv) $y = Ae^{5x} + B.e^{-5x}$

Differentitating w.r.t. x

$$\frac{dy}{dx} = A.e^{5x} \cdot 5 + Be^{-5x}(-5)$$

$$\therefore \frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$$

Again differentitating w.r.t. x

$$\frac{d^2y}{dx^2} = 5Ae^{5x} \cdot (5) - 5(-5)Be^{-5x}$$ **[1 M]**

$$= 25Ae^{5x} + 25Be^{-5x}$$

$$= 25y$$

$$\frac{d^2y}{dx^2} = 25y$$

$\therefore \frac{d^2y}{dx^2} - 25y = 0$ is the required differential equation. **[1 M]**

Topic: Differential equation_Sub Topic: Formation of Differential Equation_Level:1_ _Target-2017_XII-HSC Board (40) Test_Mathematics

(v) Let r = no of bombs hit the target

$p = 0.8$,

$q = 0.2$ $\quad (1 - p = q)$

$n = 10$ $\quad r = 4$

$p(r = 4) = {}^nC_r p^r q^{n-r} \quad r = 0, 1, 2, ..., n$

$$= {}^{10}C_4 (0.8)^4 (0.2)^6$$

$$= {}^{10}C_4 \left(\frac{8}{10}\right)^4 \left(\frac{2}{10}\right)^6$$ **[1 M]**

$$= \frac{10!}{4!6!} \times (2)^{18} \left(\frac{1}{10}\right)^{10}$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} \times (2)^{18} \times \left(\frac{1}{10}\right)^{10}$$

$$= 210 \times (2)^{18} \times \left(\frac{1}{10}\right)^{10}$$

$$= \frac{262144 \times 210}{(10)^{10}} = \frac{55050240}{(10)^{10}}$$

$$= Anti[\log 210 + 18 \log 2 - 10]$$

$$= Anti[2.3222 + 18 \log(0.3010) - 10]$$

$$= Anti(\bar{3}.7402)$$

$$= 0.0055$$ **[1 M]**

Topic: Probability_Sub Topic: Bionomial Distribution_Level: 2_Target-2017_XII-HSC Board (40) Test_Mathematics

Q. 5 (A)

(i) $\frac{dy}{dx}=\cos(x+y)$

Let $x+y=u$

$1+\frac{dy}{dx}=\frac{du}{dx}$ **[1 M]**

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}-1$

$\frac{du}{dx}-1=\cos u$

$\Rightarrow \frac{du}{dx}=1+\cos u$

$\Rightarrow \frac{du}{1+\cos u}=dx$ **[1 M]**

$\therefore$ Integrating w.r.t. x both side

$\therefore \int \frac{du}{1+\cos u}=\int dx$

$\therefore \int \frac{1}{2\cos^2 \frac{u}{2}}du = x+C$

$\therefore \frac{1}{2}\int \sec^2 \frac{u}{2}du = x+C$

$\therefore \tan\frac{u}{2}=x+C$

$\therefore \tan\left(\frac{x+y}{2}\right) = x+C$

which is the required solution of the given differential equation. **[1 M]**

Topic:Differential Equation; Sub-topic:Method of Substitution _ L-2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) Let $\int v\, dx=w$ ... (1)

then $\frac{dw}{dx}=v$ ... (2) **[1 M]**

Now, $\frac{d}{dx}(u\cdot w)=u\cdot\frac{d}{dx}(w)+w\cdot\frac{d}{dx}(u)$

$= u\cdot v + w\cdot\frac{du}{dx}$ ... From (2) **[1 M]**

By Definition of integration.

$$u \cdot w = \int\left[u \cdot v + w.\frac{du}{dx}\right]dx$$

$$= \int u \cdot v \cdot dx + \int w \cdot \frac{du}{dx}\, dx$$

$$\int u \cdot v \cdot dx = \mathrm{u} \cdot \mathrm{w} - \int w \cdot \frac{du}{dx}\, dx$$

$$= u \cdot \int v\, dx - \int\left[\frac{du}{dx}\int v \cdot \mathrm{dx}\right] dx$$ **[1 M]**

Topic:Integration; Sub-topic:Theorem of Integration by Parts_ L-1 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) $\because f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x\to 0} f(x) = f(0)$$ **[1 M]**

$$\therefore f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{e^{x^2} - \cos x}{x^2} = \lim_{x\to 0}\frac{\left(e^{x^2} - 1\right) + (1 - \cos x)}{x^2} = \lim_{x\to 0}\left(\frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{e^{x^2} - 1}{x^2} + \frac{2\sin^2\frac{x}{2}}{x^2}\right) = \lim_{x\to 0}\left[\frac{e^{x^2} - 1}{x^2} + 2\left(\frac{\sin\frac{x}{2}}{x}\right)^2\right] = \lim_{x\to 0}\left[\frac{e^{x^2} - 1}{x^2} + 2\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}} \times \frac{1}{2}\right)^2\right]$$ **[1 M]**

$$= \lim_{x\to 0}\frac{e^{x^2} - 1}{x^2} + 2 \times \frac{1}{4}\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$= 1 + \frac{1}{2}(1)^2$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

Thus, $f(0) = \frac{3}{2}$ **[1 M]**

Topic: Continuity_Sub Topic:Continuity at a Point _Level: 2_Target-2017_XII-HSC Board (40) Test_Mathematics

(B)

(i) Let δy be the increment in y corresponding to an increment δx in x.

$\therefore$ as $\delta x \to 0, \delta y \to 0$

Now y is a differentiable function of x.

$$\therefore \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$$

$$\text{Now } \frac{\delta y}{\delta x} \times \frac{\delta x}{\delta y} = 1$$ **[1 M]**

$$\therefore \frac{\delta x}{\delta y} = \frac{1}{\left(\frac{\delta y}{\delta x}\right)}$$

Taking limits on both sides as $\delta x \to 0$, we get,

$$\lim_{\delta x \to 0} \frac{\delta x}{\delta y} = \lim_{\delta x \to 0} \left[\frac{1}{\left(\frac{\delta y}{\delta x}\right)}\right] = \frac{1}{\lim_{\delta x \to 0} \frac{\delta y}{\delta x}}$$

$$\therefore \lim_{\delta y \to 0} \frac{\delta x}{\delta y} = \frac{1}{\lim_{\delta x \to 0} \frac{\delta y}{\delta x}} \quad \text{........}[as\ \delta x \to 0,\ \delta y \to 0]$$ **[1 M]**

Since limit in R.H.S. exists

$\therefore$ limit in L.H.S. also exists and we have,

$$\lim_{\delta y \to 0} \frac{\delta x}{\delta y} = \frac{dx}{dy}$$

$$\therefore \frac{dx}{dy} = \frac{1}{(dy/dx)}, \text{ where } \frac{dy}{dx} \neq 0$$ **[1 M]**

Let $y = \tan^{-1} x$

$$x = \tan y \ \Rightarrow \cos y = \frac{1}{\sqrt{1 + \tan^2 y}} = \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \sec^2 y \cdot \frac{dy}{dx} = 1 \Rightarrow \frac{dx}{dy} = \sec^2 y$$

$$\because \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{\sec^2 y} = \cos^2 y \Rightarrow \frac{dy}{dx} = \cos^2 y$$

$$\therefore \frac{d\left(\tan^{-1} x\right)}{dx} = \cos^2 y = (\cos y)^2 = \left(\frac{1}{\sqrt{1+x^2}}\right)^2$$

$$\therefore \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1 + x^2}$$ **[1 M]**

Topic:Differentiation ; Sub-Topic:Derivatives of inverse functions_L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) Here, the number of subscribers = 5000 and annual rental charges per subscriber = Rs.3000.
For every increase of 1 rupee in the rent,
one subscriber will be discontinued.
Let the rent be increased by Rs. x.
$\therefore$ New rental charges per year $= 3000 + x$
and number of subscribers after the increase in rental charges $= 5000 - x$. **[1 M]**
Let R be the annual income of the company.
Then, $R = (3000 + x)(5000 - x)$

$= 15000000 - 3000x + 5000x - x^2$ **[1 M]**

$= 1,50,00000 + 2000x - x^2$

$\therefore \frac{dR}{dx} = 2000 - 2x$ and $\frac{d^2R}{dx^2} = -2$

R is maximum if $\frac{dR}{dx} = 0$ i.e, $2000 - 2x = 0$

i.e., if $x = 1000$. **[1 M]**

$\therefore \left(\frac{d^2R}{dx^2}\right)_{x=1000} = -2 < 0$

By the second derivative test, R is maximum when $x = 1000$.
$\Rightarrow$ Thus, the annual income of the company is maximum when the annual rental charges are increased by Rs.1000. **[1 M]**

Topic:Application of Derivative; Sub-topic:Maxima and Minima _ L-2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii) $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} \cdot dx$

Let $I = \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} \cdot dx$

$$= \int_{-a}^{a} \sqrt{\frac{(a-x)(a-x)}{(a+x)(a-x)}} \cdot dx$$ **[1 M]**

$$= \int_{-a}^{a} \frac{a-x}{\sqrt{a^2-x^2}} dx$$

$$= \int_{-a}^{a} \frac{a}{\sqrt{a^2-x^2}} dx - \int_{-a}^{a} \frac{x}{\sqrt{a^2-x^2}} dx$$

$$\left[\text{but } \frac{a}{\sqrt{a^2-x^2}} \text{ is an even function and } \frac{x}{\sqrt{a^2-x^2}} \text{ is an odd function}\right]$$

$$= 2a \cdot \int_{0}^{a} \frac{1}{\sqrt{a^2-x^2}} dx - 0$$ **[1 M]**

$$= 2a \cdot \left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= 2a \cdot \left[\sin^{-1}1 - \sin^{-1}0\right]$$

$$= 2a\left[\frac{\pi}{2} - 0\right]$$

$$\therefore \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} \cdot dx = \pi a$$ **[1 M]**

Topic:Definite Integration; Sub-topic:Property_ L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

Q. 6 (A)

(i) $f(0) = 1........(given).....(1)$

for $x > 0$, $|x| = x$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{|x|}$$

$$= \lim_{x\to 0^+} \frac{x}{x}$$

$$= \lim_{x\to 0^+} (1) \qquad .....[x \to 0, x \neq 0]$$ **[1 M]**

$= 1$

for $x < 0, |x| = -x$

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x|}$$

$$= \lim_{x\to 0^-} \frac{-x}{x}$$

$$= \lim_{x\to 0^-} (-1) \qquad .....[x \to 0, x \neq 0]$$

$= -1$ **[1 M]**

$$\therefore \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$$

$\therefore f$ is discontinuous at $x = 0$

Here $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist

hence, it is discontinuous at $x = 0$ **[1 M]**

Topic:Continuity; Sub-topic:Continuity at a Point _ L-2_Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) Let P be the population of the country at time t.

Given : $\frac{dP}{dt} \propto P$ $\therefore \frac{dP}{dt} = kP$ (where k is a constant)

$\therefore \frac{1}{P} dP = kdt$

Integrating, both side w.r.t. x

$\int \frac{1}{P} dP = k\int 1dt + c$ **[1 M]**

$\therefore \log P = kt + c$

$\therefore P = e^{kt+c} = e^{kt} \cdot e^{c}$

Let $e^{c} = \alpha$

$\therefore P = \alpha \cdot e^{kt}$

Let initial population at t = 0

$\therefore N = \alpha \cdot e^{0}$ $\therefore N = \alpha$

$\therefore \quad P = N \cdot e^{kt}$

Given P = 2N when t = 60 years,

$\therefore 2N = N \cdot e^{60k}$

$\therefore 2 = e^{60k} \quad \Rightarrow \quad k = \frac{1}{60}\log 2$

$\therefore \quad P = N \cdot e^{60k}$ **[1 M]**

Required t when P = 3N

$\therefore 3 = e^{kt} \Rightarrow \log 3 = kt$

$\therefore \log 3 = \left(\frac{1}{60}\log 2\right) \cdot t$

$\therefore \quad t = \frac{60\log 3}{\log 2}$

$= \frac{60 \times 1.0986}{0.6912}$

= 95.4 years (approx.)

$\therefore$ The population of the countr will triple approximately in 95.4 years. **[1 M]**

Topic:Differential Equations; Sub-topic:Application of Differential Equation_ L-3 _Target-2017_XII-HSC Board (40) Test_Mathematics

(iii)

(a) Let X = number of heads

p = probability of getting head

$\therefore p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given : $n = 8$

$\therefore X \sim B\left(8, \frac{1}{2}\right)$

The p.m.f. of X is given as

$P(X = x) = P(x) = {}^{n}C_{x} p^{x} q^{n-x}$

$P(X) = {}^{8}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{8-x}, \quad x = 0, 1, 2......8$ **[1 M]**

$P(exactly\, 5\, heads) = P[X = 5]$

$= P(5) = {}^{8}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{8-5}$

$= {}^{8}C_{3}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{3}$ $\left[\because {}^{n}C_{x} = {}^{n}C_{n-x}\right]$

$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{1}{256} = \frac{7}{32}$

$\therefore P[X = 5] = 0.21875$

Hence, the probability of getting exactly 5 heads is 0.21875. **[1 M]**

(b) $P(getting\ heads\ at\ least\ once)$

$= P[X \geq 1] = 1 - P[X = 0]$

$= 1 - p(0) = 1 - {}^{8}C_{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{8-0}$

$= 1 - \left(\frac{1}{2}\right)^{8} = 1 - \frac{1}{256} = \frac{255}{256}$

$\therefore P[X \geq 1] = 0.996$ **[1 M]**

Hence, the probability of getting heads at least once is 0.996.

Topic: Binomial Distribution_Sub Topic:Bernaulis Trial _Level: 2__Target-2017_XII-HSC Board (40) Test_Mathematics

Q. 6 (B)

(i) $$I=\int\frac{d\theta}{\sin\theta+2\sin\theta\cos\theta}$$

$$=\int\frac{d\theta}{\sin\theta\left(1+2\cos\theta\right)}$$

$$=\int\frac{\sin\theta\,d\theta}{\sin^2\theta\left(1+2\cos\theta\right)}$$ **[1 M]**

$$=\int\frac{\sin\theta\,d\theta}{\left(1-\cos^2\theta\right)\left(1+2\cos\theta\right)}=\int\frac{\sin\theta\;d\theta}{\left(1-\cos\theta\right)\left(1+\cos\theta\right)\left(1+2\cos\theta\right)}$$

Let $\cos\theta=t\quad\therefore -\sin\theta\,d\theta=dt$

$$\therefore I=\int\frac{-dt}{\left(1-t\right)\left(1+t\right)\left(1+2t\right)}$$ **[1 M]**

Let $\dfrac{-1}{\left(1-t\right)\left(1-t\right)\left(1+2t\right)}=\dfrac{A}{1-t}+\dfrac{B}{1+t}+\dfrac{C}{1+2t}$ ....(1)

$\therefore -1=A\left(1+t\right)\left(1+2t\right)+B\left(1-t\right)\left(1+2t\right)+C\left(1-t^2\right)$ ....(2)

put $t=1$, in equation (2) $\quad -1=A(2)(3)\quad\therefore A=-\dfrac{1}{6}$

put $t=-1$; in equation (2) $\quad -1=B(2)(-1)\quad\therefore B=\dfrac{1}{2}$

put $t=-\dfrac{1}{2}$ in equation (2) $\quad\therefore -1=C\left(\dfrac{3}{4}\right)\quad\therefore C=-\dfrac{4}{3}$ **[1 M]**

Put value of A, B, C in equation (1)

$$\therefore I=\int\frac{\left(-\frac{1}{6}\right)dt}{1-t}+\int\frac{\left(\frac{1}{2}\right)dt}{1+t}+\int\frac{\left(-\frac{4}{3}\right)dt}{\left(1+2t\right)}$$

$$=\left(\frac{-1}{6}\right)\frac{\log|1-t|}{-1}+\frac{1}{2}\log|1+t|-\left(\frac{4}{3}\right)\frac{\log|1+2t|}{2}+c$$

$$=\frac{1}{6}\log|1-t|+\frac{1}{2}\log|1+t|-\frac{2}{3}\log|1+2t|+C$$

$$=\frac{1}{6}\Big[\log|1-t|+3\log|1+t|-4\log|1+2t|\Big]+C$$

$$=\frac{1}{6}\Big[\log|1-\cos x|+3\log|1+\cos x|-4\log|1+2\cos x|\Big]+C$$ **[1 M]**

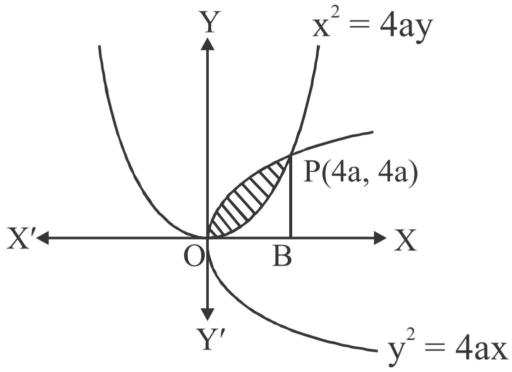
Topic:Integral; Sub-topic:Partial Fraction_ L-2 _Target-2017_XII-HSC Board (40) Test_Mathematics

(ii) The equations of the parabolas are

$y^2 = 4ax$ ... (i)

and $x^2 = 4ay$ ... (ii)

$\therefore \left[\dfrac{x^2}{4a}\right]^2 = 4ax$ by... (ii)



[1 M]

$x^4 = 64a^3x$

$x\left[x^3 - (4a)^3\right] = 0$

x = 0 and x = 4a

$\therefore$ y = 0 and y = 4a **[1 M]**

$\therefore$ The points of intersection of curves are O(0, 0), P(4a, 4a)

$\therefore$ The required ares is, A = (Area under parabola $y^2 = 4ax$) – (Area under parabola $x^2 = 4ay$)

$$= \int_0^{4a} \sqrt{4 \cdot ax}\, dx - \int_0^{4a} \frac{x^2}{4a}\, dx$$ **[1 M]**

$$= \sqrt{4a} \cdot \frac{2}{3}\left[x^{3/2}\right]_0^{4a} - \frac{1}{4a} \cdot \frac{1}{3}\left[x^3\right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3} \times 4a\sqrt{4a} - \frac{1}{12a} \times 64a^3$$

$$= \frac{32}{3}a^2 - \frac{16}{3}a^2$$

$$= \frac{16}{3}a^2 \text{ sq.units}$$ **[1 M]**

Topic:Definite Integral; Sub-topic:Area between two curves L-3___XII-HSC Board (40) Test_Mathematics

(iii) c.d.f. of X is given by

$$F(x) = \int_{-1}^{x} f(y)\, dy$$

$$= \int_{-1}^{x} \frac{y^2}{3} dy = \left[\frac{y^3}{9}\right]_{-1}^{x}$$

$$= \frac{x^3}{9} + \frac{1}{9}$$

Thus $F(x) = \dfrac{x^3}{9} + \dfrac{1}{9}, \quad \forall\, x \in R$ **[1 M]**

Consider $P(X < 1) = F(1) = \dfrac{(1)^3}{9} + \dfrac{1}{9} = \dfrac{2}{9}$ **[1 M]**

$P(X \le -2) = 0$ ($\therefore$ range of X is (–1, 2))

$P(X > 0) = 1 - P(X \le 0)$

= 1 – F(0)

$= 1 - \left(\frac{0}{9} + \frac{1}{9}\right)$

$= \frac{8}{9}$ **[1 M]**

$P(1 < X < 2) = F(2) - F(1)$

$= \left[\frac{8}{9} + \frac{1}{9}\right] - \left[\frac{1}{9} + \frac{1}{9}\right] = 1 - \left[\frac{2}{9}\right]$ **[1 M]**

$= \frac{7}{9}$

Topic:Probability Distribution; Sub-topic:p.d.f. L-2__XII-HSC Board (40) Test_Mathematics